With a motor included, we have a modified condition for stability: the rate of change of total system energy must be of the same sign as $\dot{\theta}$. This rate is the sum of the rate of change of kinetic energy and the rate of motor energy "transfer," the latter being positive if energy is removed by the motor. Neglecting friction, the spin rate of the body not containing the damper can be changed only by the application of motor torque. The rate at which energy is transferred is the product of this torque and the relative spin rate. Again, suppose that body I is rigid. Then the motor torque is equal to $C_1\dot{\omega}_{1Z}$, and the rate at which energy is supplied or removed by the motor ($\dot{T}_M$) is given by

$$\dot{T}_M = C_1\dot{\omega}_{1Z}(\omega_{2Z} - \omega_{1Z}) \tag{7}$$

Note that $\dot{T}_M \neq C_2\dot{\omega}_{2Z}(\omega_{2Z} - \omega_{1Z})$ in magnitude, as body II is also losing (or gaining) spin momentum because of the presence of the damper.

Assume that $\omega_{2Z} > \omega_{1Z}$. Then, if $\dot{\omega}_{1Z}$ is positive, there is a net reduction in kinetic energy due to the action of the motor. Hence, the total rate of change of system energy is the sum of the rates given by Eqs. (3) and (7) and the condition for stability again reduces to that given by Eq. (5) for the no-motor case. If body II is rigid,

$$\dot{T}_M = C_2\dot{\omega}_{2Z}(\omega_{2Z} - \omega_{1Z}) \tag{8}$$

and, for $\omega_{2Z} > \dot{\omega}_{1Z}$, kinetic energy is increased for positive $\dot{\omega}_{2Z}$. The rate of change of total system energy can be obtained in this case by taking the difference of Eqs. (4) and (8), which leads to a condition for stability given by Eq. (6). We conclude, therefore, that the presence of the motor does not in any way affect the result and that the conditions for stability in Eqs. (5) and (6) depend only on the location of the damper. If one body is stationary or rotating very slowly compared with the other, the conditions reduce to those given earlier in the paper.

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Alouette Topside Sounder Satellite: Experiments, Data, and Results

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Introduction

THE Alouette satellite (1962 Beta Alpha One) was launched 0605 GMT September 29, 1962, from Vandenburg Air Force Base, Calif. into a nearly circular orbit with an inclination of 80.5° and 1000 km above the earth. The satellite was designed and constructed in Canada at the Defence Research Telecommunications Establishment and launched into orbit on a Thor-Agena B.

In the first ten months of its operation, the satellite received and executed over 10,000 commands in 2000 hr of operation of the four experiments aboard. In this time, there were no equipment failures. The sole observable degradation was in reduced power output (to 63% of the output at launch) of the solar cells due largely to energetic particle damage from the artificial radiation belt produced by the Starfish test of July 9, 1962.

Figure 1 shows the Alouette spacecraft ready for launch. It is a flattened sphere in shape, covered with about 6500 solar cells, and weighs 319 lb. Its most distinguishing feature is the pair of crossed antennas, one 150 ft and the other 75 ft from tip to tip. Designed with reliability as a prime consideration, the spacecraft has been described in detail elsewhere.[2,12]

Alouette carries three novel experiments, a sweep-frequency (0.5 to 11.5 Mc/sec) topside sounder, a receiver (part of the sounder) capable of monitoring cosmic radio noise, and a very-low-frequency (VLF) receiver. A group of six counters for energetic particles makes the fourth experiment. This paper will discuss these four experiments and summarize briefly some results obtained.

Topside Sounder

The term "topside sounder" has been coined to describe an experiment in which an ionosonde is placed above the F-layer peak of the ionosphere. Figure 2 shows diagramatically the electron number density distribution below 1000 km and the complementary fashion in which a topside and a bottomside ionosonde can be used to determine this distribution. (An ionosonde measures echo range of a radio pulse of appropriate frequency.)

The Alouette topside sounder enables the topside distribution to be measured, under favourable conditions, approximately every 100 km when the satellite is operating. The NASA Minitrack chain of stations in North and South America, along with telemetry stations in Canada, and the South Atlantic station operated by Great Britain, permit measurements to be made near the 75°W meridian nearly continuously from 80°N to 80°S latitude. After a somewhat lengthy analysis procedure, a cross section of the ionosphere, as shown in Fig. 3, can be produced.

Figure 3 shows contours of constant electron density, in megacycles, as a function of height and latitude. It also demonstrates the very comprehensive measurement of the ionosphere in depth which is possible from a topside sounder satellite.

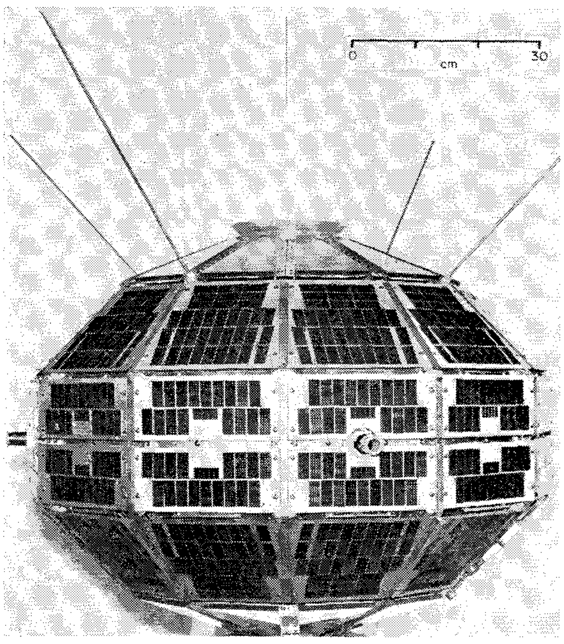

Fig. 1 Alouette spacecraft with sounding antennas retracted in launch position (after Molozzi).

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Table 1 Energetic particle counter characteristics

Detector	Particle response Electrons	Protons	α particles	Remarks
302 Geiger	3.9 Mev 50% trans.	>33 Mev	...	Omnidirectional
223 Geiger	>40 kev	>500 kev	...	Directional
223 Geiger with Mag. Field	>250 kev	>500 kev	...	Directional
Silicon junction	...	1.3–7 Mev	4.3–28 Mev	
Geiger telescope	...	>100 Mev	>400 Mev	
Plastic scintillator	...	>100 Mev	>400 Mev	

From a series of such diagrams, the development of the equatorial ionosphere can be readily followed. For example, in the forenoon, the contours of ionization form a dome over the magnetic equator, gradually rising as noon is approached, e.g., the 2 Mc/sec contour rising from 450 to 750 km. This may represent an actual expansion of the atmosphere under the influence of solar heating. In the afternoon, the ionization appears to settle along successive magnetic field lines. The ionization appears to slide down the field lines, piling up at about ±20°N and S magnetic latitude to form the well-known geomagnetic anomaly. This process has been described recently by Lockwood and Nelms.[6] The limiting field line is sharply defined by a "ledge" of ionization, which can be traced as shown by the points plotted near a calculated field line.

In the midlatitude region, the iso-ionic contours are relatively level and vary slowly and regularly. The temperate ionosphere is relatively well behaved.

The high latitude ionosphere (above 45° in Fig. 3) shows abrupt horizontal and vertical changes. Perhaps coincidentally, the "trough" or depression seen near 45°N is located close to the region of dumping of the flux of energetic particles in the radiation belt. At latitudes above about 45°, turbulent ionization giving rise to diffuse echoes on the ionosonde is seen regularly. Inside the auroral zone, vertical sheets of irregular ionization a few hundred kilometers thick are observed on the ionosonde as the satellite approaches and flies through them. This is a brief summary of the extensive results of the analysis of measurements from the topside sounder as summarized from the literature.[3, 7]

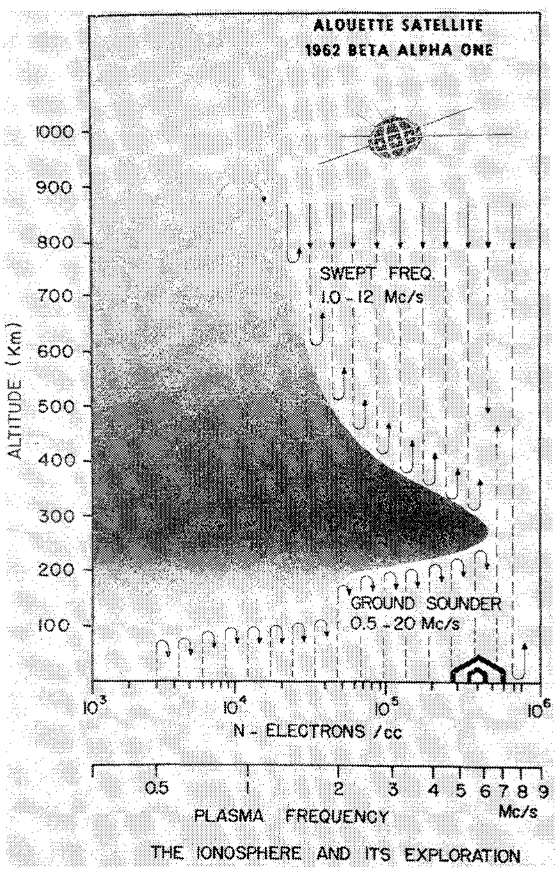

Fig. 2 Representation of electron distribution in the ionosphere measured by topside and bottomside soundings (after Molozzi).

Cosmic Radio Noise Experiment

The receiver in the ionosonde has been designed with a gain-control loop (AGC) in order to maintain a constant receiver output to the telemetry system. The AGC voltage is telemetered to the ground, making possible relative measurements of the cosmic radio background as the ionosonde sweeps in frequency from 0.5 to 11.5 Mc/sec.

At low frequencies, cosmic radio noise is prevented from reaching the receiver in the satellite by the presence of the ionosphere. At frequencies between the plasma frequency at the satellite and the maximum frequency of the *F* layer, cosmic radio noise reaches the antenna from above. At frequencies above the critical penetration frequency of the *F* layer, radiations from below the ionosphere, largely artificial, reach the receiver. The recordings in the center portion of the frequency band can therefore be used in studies of cosmic radio noise.

Hartz has carried out studies of these data.[4, 5] He finds that the spectral index in the 2 to 4 Mc/sec band is approximately 1.8. He has also examined the noise power at a frequency of 2.0 Mc/sec as a function of the area of the sky being observed. The center of the galaxy, which is a region of increased radiation at higher frequencies, does not appear to radiate as much at 2 Mc/sec. This is interpreted as evidence of absorption of 2 Mc/sec radio waves by hydrogen in the galaxy.

VLF Receiver

First results of the analysis of data from the VLF receiver on board Alouette have been published by Barrington and Belrose.[1] They report that whistlers are readily received at the satellite, the signals being very strong. Short "fractional hop whistlers," which have made one traverse of the ionosphere, are dispersed primarily at the low-frequency (below 2 kc/sec) end of the band (700 cycles/sec to 10 kc/sec). The signal-to-noise ratio is sufficiently high that the dispersion caused by heavy ions can be detected, and a value for the mean ion mass calculated.

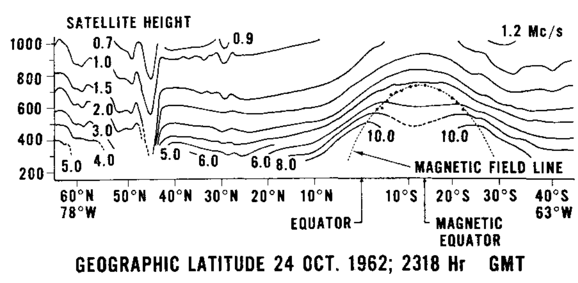

Fig. 3 A cross section of the ionosphere between 65° N and 45° S latitude at 75°W longitude. Contours are of constant plasma frequency in Mc/sec.

The spectrograms from whistlers observed by the Alouette satellite are very similar to those observed on the ground. This is perhaps unexpected, as whistlers were thought to be propagated strictly along magnetic field lines, only one of which would be intercepted by the satellite.

Broad bands of VLF noise, hiss, drawn chorus, etc., are also observed. These appear in a highly variable manner but do show evidence of magnetic field control. On the assumption that these emissions are caused by energetic particles whose velocity along the magnetic field approximately equals the velocity of propagation of VLF radio waves, the component of velocity along this field is appropriate to electrons with energies of the order of a few kiloelectron volts.

Energetic Particle Experiment

The six energetic particle counters contained in Alouette have characteristics as shown in Table 1.

Studies of the results of the first few months data from these counters have been carried out by McDiarmid et al.[8–11] These authors have discussed the observations made of the artificial radiation belts established by the "Starfish" test on July 9, 1962, and the Russian high-altitude nuclear tests in October and November of that year.[9] On one test, Alouette observed the initiation of a short-lived belt of electrons produced by one of the Russian tests.[10] The electrons were observed over Australia shortly after the test, but presumably they were dumped in the South Atlantic anomaly in the first circuit of the earth, as they were not observed on subsequent orbits.

One of the most interesting observations concerns the high flux of particles being dumped from the natural radiation zones at high latitudes. The energy input to the ionosphere from this source is significant, and it is reasonable to suppose that a number of ionospheric effects are due to this flux.[8, 11]

Conclusions

The experiments in the Alouette satellite have been most successful. A number of new results have been obtained, some of which lead to an appreciation of the importance of magnetic control of ionization in the ionosphere and the importance of the energy from the energetic particles dumped into the ionosphere from the radiation belts.

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Application of Biot's Variational Method to Convective Heating of a Slab

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Introduction

Biot, in his paper[1] on the application of the variational principle to transient heat conduction problems, has discussed in detail examples involving prescribed surface temperatures. Subsequently, Lardner[2] applied Biot's method to problems for which the heat flux is prescribed. In this paper, we consider a semi-infinite slab and extend Biot's method to problems with a coupled convective boundary condition of the form

$$U(\theta_0 - \theta) = -k(\partial\theta/\partial x) \tag{1}$$

at $x = 0$. Here x is the distance of a point in the semi-infinite slab from its surface, U the heat-transfer coefficient of the convection stream, θ_0 the temperature of the stream (assumed constant), k the thermal conductivity of the slab, and θ is the temperature in the slab (see Fig. 1). Models of this type have important applications to many convection heating problems, such as that one encountered in rocket engine nozzles where heat is transferred convectively from the hot gas stream to the nozzle wall.

Rough Approximation and Its Comparison with the Exact Solution

The approximate solution[1] for a semi-infinite slab with a prescribed boundary temperature θ_0 is

$$\theta = \theta_0[1 - (x/q_1)]^2 \qquad q_1 = 3.36(kt/c_v)^{1/2} \tag{2}$$

The penetration depth q_1 is a generalized coordinate in the variational formulation used by Biot. For the convection condition of the present example, Eq. (1), the temperature at $x = 0$ is θ_1, which starts from zero at $t = 0$, then increases and eventually reaches θ_0 at $t = \infty$. A naturally convenient first approximation suggested by Eq. (2) is

$$\theta = \theta_1[1 - (x/q_1)]^2 \qquad q_1 = 3.36(kt/c_v)^{1/2} \tag{3}$$

where the new unknown temperature θ_1 in Eq. (3) may be determined from the boundary condition, Eq. (1). The result is

$$\theta_1 = \theta_0[1 + (2k/Uq_1)]^{-1} \tag{4}$$

The approximation used for Eq. (3) implies that the growth of q_1 is not affected by the variation of temperature at $x = 0$. For large times, when the time rate of increase of θ_1 is no longer appreciable, this assumption will be physically reasonable. However, the assumption is certainly inappropriate for small times. Since condition (1) tends to that of the prescribed boundary temperature as $U \rightarrow \infty$, approximation (3) is expected to be good for large U. To determine the

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